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Upscaling of Relative Permeability Curves for Reservoir Simulation: An Extension to Areal Simulations Based on Realistic Average Water Saturations

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Abstract

Special core analysis laboratories generally obtain relative permeability curves through frontal advance theory developments. The outer face of the sample, where production rates are measured, is usually selected to perform calculations. As a result, relative permeability curves are reported as function of point fluids saturations and not as function of average saturations. In spite of this procedure, numerical simulators perform their calculations using the average water saturation within every cell. Although point and average saturations are expected to be the same at infinitely small grid sizes, this is not the case with real application grids.

As previously shown by the authors[1], relative permeability curves as a function of average water saturation could be used to overcome numerical dispersion and drastically reduce the number of grid cells needed for linear numerical simulation situations.

In this paper a series of experimental tests were performed to extend the methodology to 2D (areal) displacement. On this geometry, an additional issue should be taken into account that does not impact the linear cases. A geometry effect appears as a consequence of flow leaving the grid cells in a direction perpendicular from where it enters. As in the previously studied linear situation, water-cut as a function of produced oil was analyzed. The history was matched using a numerical simulator and relative permeability curves defined as a function of point saturations and average water saturation for different grid sizes. Representative relative permeability curves were previously determined through routine laboratory explicit computation based on linear unsteady state displacement. It is shown that only curves taking into account average saturation values and geometry considerations give reliable data, and a general methodology to obtain the proper relative permeability curve for every cell is examined.

Introduction

Relative permeability curves are of extreme importance to reservoir evaluations due to their ability to predict fluid production during reservoir exploitation. They establish, for any particular phase, a functional dependence between phase saturation and the rock's ability to produce.

These curves are determined in special core analysis laboratories through a sequence of standard measurements and calculations generally performed using some adaptations of frontal advance theory[2-3]. Conceptually, this theory[3-5] requires pressure gradients and fluid saturations at the production face of the sample, leading to a set of calculated relative permeability values that are function of point saturation (saturation at the outer face of the porous sample).

A different approach is postulated by numerical simulation. CVFD simulators[6] postulate the continuity or mass equation on a discrete volume basis. This concept determines that multiphase fluid movement at in-flow or out-flow boundaries (cell to cell limit, injector wells or producer wells) is in direct relationship to the average fluid saturations of every grid cell.

As previously shown by the authors[1], point saturation may differ markedly from average saturation, resulting in misinterpretation and leading to erroneous upscaling results. Based on this difference, numerical simulators may give inadequate answers if laboratory relative permeability curves are used directly.

This situation was fully analyzed in a previous paper[6] for 1D geometry. In this paper, an overall discussion of the solution for the 2D situation is presented. Also, a more general approach is presented in **Appendix I**, where the fundamentals of relative permeability concepts are discussed.

As shown here, no simple answers are available for geometries other than 1D, so this paper focuses on a critical discussion of the accepted methodology for upscaling that is being used at the present.

Problem Description

To obtain a better answer through numerical simulation, curves describing the ability to produce or to admit fluids as a function of average fluid saturation are needed.

In 1D problems, a simple methodology has been presented[1] to tackle the determination of realistic curves from standard relative permeability data. But in 2D (areal) geometries a new issue appears. In this type of geometry, the flow diverts from a linear system causing some geometric effects to appear.

Currently, there is no way to take these geometric effects into account as a coefficient because they are not routinely determined at lab scale, where only 1D displacements are routinely performed.

1D Solution

Based on the sequence of standard measurements performed at special core analysis laboratories and through basic mathematics, the following can be computed:

1. ***The average water saturation of the sample***. As long as oil production is recorded at every time during the test, it is possible to determine the average water saturation of the sample by performing a material balance.
2. ***The fluid production rates***. By fitting the measured fluid volumes produced against time (V_o vs. t and V_t vs. t) the rates of the producing fluids can be obtained on a time basis. Physically, these rates correspond to the ones at the end or outer face of the plug.

In this way, Darcy formulation for linear systems can be applied using the available data to obtain relative permeabilities values using the following equations:

$$kr_o = \frac{q_o \cdot A}{k \cdot \mu_o} \cdot \left(\frac{\Delta P}{L} \right) \qquad (1)$$

$$kr_w = \frac{q_w \cdot A}{k \cdot \mu_w} \cdot \left(\frac{\Delta P}{L} \right) \qquad (2)$$

where ΔP is assumed to be constant owing to negligible capillary forces.

The resulting curves from equations 1 and 2 combined with the average water saturation of the sample results in a set of relative permeability curves, called "average" relative permeabilities.

Figs. 1 and **2** sketch the relationship between average water saturation and point water saturation in a linear system of three grid cells.

Due to the independence of Welge equations to system length, the first cell in a system of many grid cells maintains the same relationship among its variables as the relationship that exists in a one-cell system. This is similar to the material balance case in which the relative permeability curves versus average water saturation always describes the relationship between water saturation of the cell (average) and its point production.

On the other hand, as shown in **Fig 2**, as the number of grid cells between the injection and production site increases, the average grid cell water saturation and the point saturation at the production location become more similar. So the point saturation curve computed using the frontal advance theory adequately describes the last cell of a multi-cell system.

An adequate linear combination of the two types of relative permeability curves can reasonably represent the behavior of the intermediate cells. It was previously shown by the authors[1] that excellent agreement between the simulated and measured data validated the proposed methodology. Another advantage is that this type of mixing rule is easily programmable for automatic computation in numerical simulators.

2D Case

Two extreme situations that may occur during areal displacements are depicted on **Figs. 3** and **4**.

Fig. 3 shows the situation for a rectangular cell where displacement characteristics are very similar to those for 1 D displacements. In this situation, the inflow and the outflow are in parallel boundaries of the cell. The 1D solution previously reported appears to be adequate when fluid flow follows this geometry.

Fig. 4 shows a very different situation. For this particular case, the flow leaves the cell in a perpendicular direction from where it enters. A shortcut between inflow limit and outflow limit is expected to appear on the corner labeled as "S".

In these figures, if both cells are assumed to be saturated with oil at Sw_{irr} condition, and water is injected from the border identified as "A", two different situations arise:

- Flow geometry in **Fig. 3** implies that a certain amount of water must be injected before some water could be produced at "B". The average water saturation must be greater than Sw_{irr} at breakthrough.
- On the other hand, flow geometry in **Fig. 4** causes water to be produced at border "C" from nearly the beginning of the water injection, making the average water saturation of the cell not necessarily greater than Sw_{irr} at breakthrough.

This brief analysis leads to the conclusion that the curve relating fluid production to average cell saturation is adequate for the situation depicted in **Fig. 3** but is unable to describe the situation depicted in **Fig. 4**.

As a consequence, the curve relating fluid production with average fluid saturation is not only dependant on fluid-rock characteristics and distance from injectors and producers (as shown in the 1D case), but also on geometric factors related to preferential stream lines.

Experimental Data

A synthetic porous media was constructed in the laboratory to perform fluid displacement tests in 1D and 2D geometries.

First, a 1D displacement test was conducted to determine relative permeability curves. Point and average saturation relative permeability curves are shown in **Fig. 5** and **6** respectively.

Afterwards, a 2D displacement test resembling a quarter of five spot geometry was used to generate measured data to be matched in the numerical simulation stage.

Numerical Simulation Results

Three main scenarios were considered. These scenarios include the sensitivity to grid cell number, different types of relative permeability curves and grid geometries.

Scenario 1 - Number of Grid Cells

In a first stage, a rectangular geometry with a 3x3 grid using the standard laboratory relative permeability curves (point water saturation at the end face of the sample) was run. In this case the injector well and producer well were located at opposite corners making the resulting grid a rotated one as shown in **Fig. 7**. After that, other cases with an increasing number of grid cells were performed.

The evolution of the produced water cut for the 3x3 case run against dimensionless cumulative production (*Np*/*N*) is presented in **Fig. 8**. In this figure, the results of 9x9 and 73x73 grids as long as the measured data are also plotted for comparison. It is clearly seen that the 3x3 grid gives a bad match and it becomes worse as the number of grid cells increases.

Scenario 2 - Dake´s Approach

L. Dake[7] proposed an empirical partial solution to the numerical dispersion problem by generating step functions in the saturation range not defined by the frontal advance theory of Buckley and Leverett[3].

Using the 3x3 geometry shown in Fig. 7, the point saturation relative permeability curve and the average saturation relative permeability curve were tested. Also a combination of both types of relative permeability curve was tested.

As expected, the curves presented in **Fig. 9** reduce the amount of numerical dispersion observed before, but it also shows that the breakthrough time is increased in comparison to measured data. None of the solutions presented here could be considered a good match.

Scenario 3 – Grid Geometry

A change in the grid orientation was proposed and a non-rotated geometry, as shown in **Fig. 10**, was analyzed.

Non-rotated grids of 3x3 and 9x9 were analyzed and the results are presented in **Fig. 11** along with the results of the 3x3 grid from Scenario 1 and the measured data. It can be seen that the non-rotated grid shows the same trend as the rotated grid as long as the number of cells increases. Furthermore, the difference between orientations vanishes as the number of cells between the injector well and the producer well increases. As in the previous scenarios, a good match could not be obtained.

Conclusions

1. The conventional numerical simulation approach did not honor the physics of the areal lab model. Some kind of areal alteration should be applied to obtain an adequate description of the physical system.
2. Only on a 1D system the previously presented methodology of relative permeability upscaling does honor the physics of a fluid flow dominated by viscous forces while it cancels out numerical dispersion.
3. Changes in grid cell number, types of relative permeability curves and grid geometries were analyzed. None of the run models achieved a good match between the measured and simulated data.
4. The streamline simulation approach reduces the dimensionality of the areal problem to a 1D system. As this system has already been solved, streamline-based simulation presents an advantage over conventional finite difference simulation.
5. The grid orientation effect disappears as the number of grid cells increases.

Nomenclature

A = Area, cm^2.
k = Absolute permeability, darcys.
kr_o = Oil relative permeability.
kr_w = Water relative permeability.
L = Length, cm.
N = Original oil in place, m^3.
Np = Cumulative oil produced. m^3.
q_o = Oil rate, cm^3/s.
q_w = Water rate, cm^3/s.
Sw_{irr} = Irreducible water saturation, %.
V_o = Cumulative volume of oil produced, cm^3.
V_t = Cumulative volume of total fluid produced, cm^3.
ΔP = Pressure difference, atm.
μ_o = Oil viscosity, cp.
μ_w = Water viscosity, cp.

References

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Figures

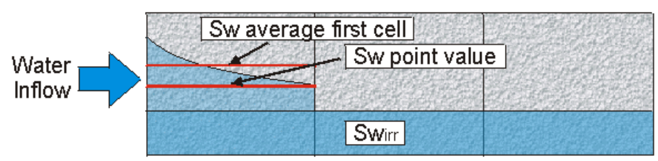

Figure 1 – Point and average water saturation for cell 1.

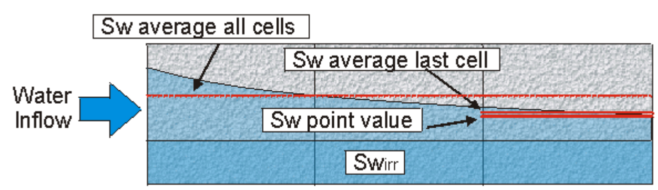

Figure 2 – Point and average water saturation for cell 3.

Figures – cont.

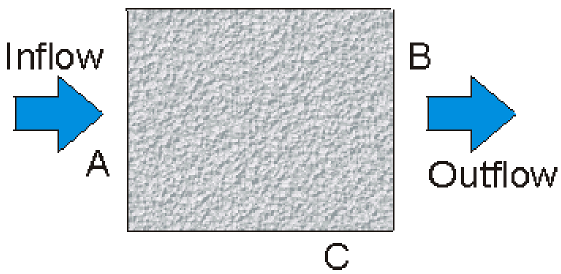

Figure 3 – Linear flow geometry between Inflow & Outflow.

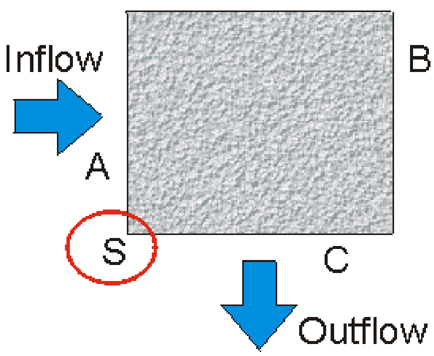

Figure 4 – Perpendicular flow geometry between Inflow & Outflow.

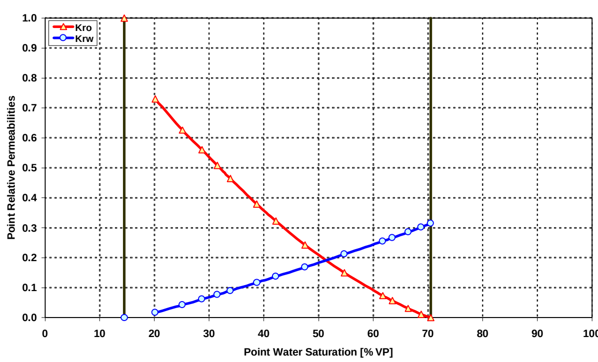

Figure 5 – "Point Saturation" relative permeability curve.

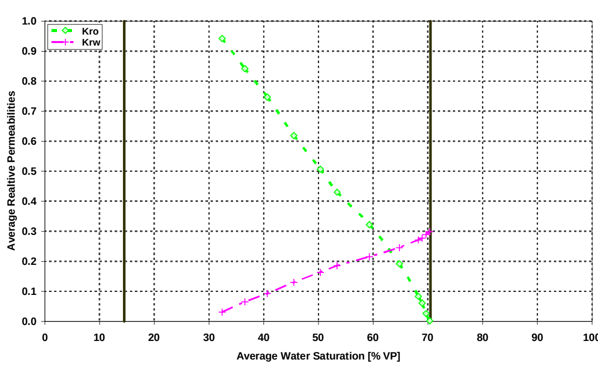

Figure 6 – "Average Saturation" relative permeability curve.

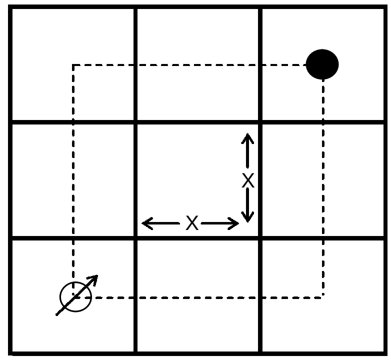

Figure 7 – Rotated grid sketch.

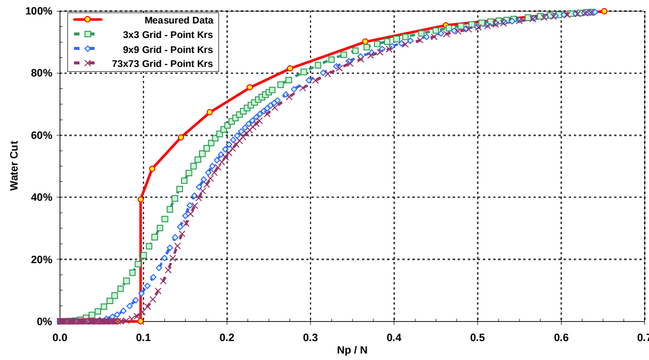

Figure 8 – Effect of number of cells on the results – point krs.

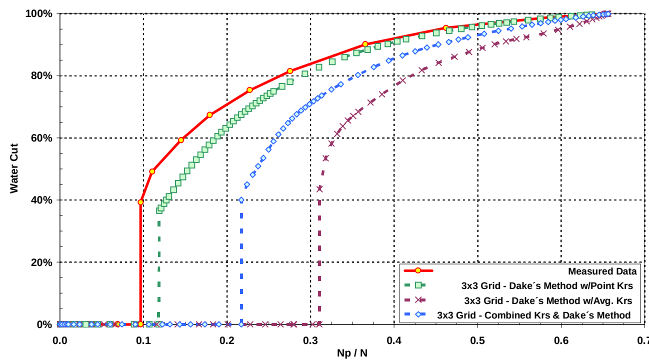

Figure 9 – Effects of different krs methodologies on the results.

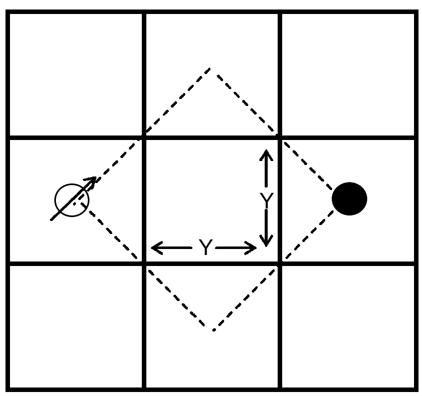

Figure 10 – Non-Rotated grid sketch.

Figures – cont.

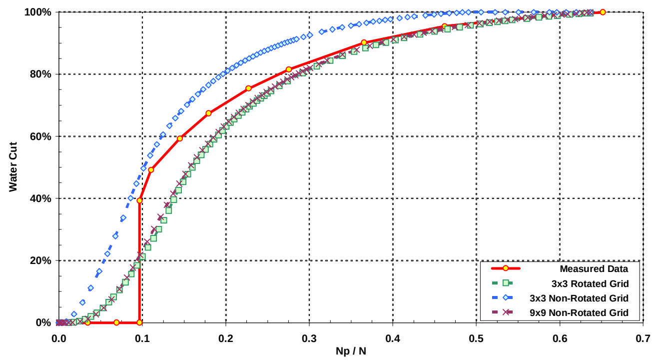

Figure 11 – Effects of grid orientation on the results.

Appendix I – Darcy and Mutiphase flow.

Conceptual Analysis

Darcy´s law (Eq. 3) is used to describe one-phase flow through linear porous materials. This law, which relates flow velocity with pressure gradient, has three components:

- A geometric factor given by length and area of the porous medium.
- A fluid-related factor (viscosity).
- A property of the porous material (Permeability).

$$Q = K \cdot A \cdot \Delta P / (\mu \cdot L) \quad \text{...(3)}$$

So, permeability is usually defined as "the ability of a porous material to **conduct** fluids".

Experimentally, only **injection** rate or **production** rate can be measured. But during the flow of an uncompressible homogeneous fluid, the rate of **conduction** equals **injection** rate and **production** rate. So, measuring only one of these rates is enough to obtain the value of the others.

For multiphase fluid flow, Darcy´s law is usually "corrected" using a different factor for each flowing phase. This factor is known as relative permeability curve.

$$Qw = K \cdot Krw \cdot A \cdot \Delta Pw / (\mu w \cdot L) \quad \text{................................ (4)}$$

$$Qo = K \cdot Kro \cdot A \cdot \Delta Po / (\mu o \cdot L) \quad \text{................................. (5)}$$

Relative permeability has different values for every fluid saturation. However, a new problem arises during unsteady multiphase fluid flow.

In multiphase fluid flow the rate of **conduction** for each phase loses its equivalence with **injection** rate, and **production** rate. When more than one phase is flowing, it is not possible to measure **production** rate or **injection** rate to determine the **conduction** rate of each phase.

Also Darcy´s law, applied to each phase, is based on **conduction** rate exclusively.

Two ways were found to solve this "inconsistency". The first one is during measurement and the other during calculation.

- Experimentally a method was developed to re-create Darcy´s requirements (**Injection** = **Conduction** = **Production**). This methodology was known as "steady state", where simultaneous **injection** rate is maintained until the **production** rate equals the **injection** rate of each phase.
- Through calculation, equations were solved during unsteady state displacement to obtain values in a dimensionless section. This is known as "unsteady state" or Welge methodology.

Briefly, to allow the use of Darcy´s law during multiphase fluid flow, it was necessary to generate a unique saturation at the position of calculation. In this way, once again:

Injection = Conduction = Production

- During "steady-state" measurements the whole sample has the same saturation.
- Using "unsteady-state" methodology, all calculations are made in a single point (the outlet face) where the saturation is unique (point saturation).

Both methodologies ("steady-state" and "unsteady-state") give the same result when applied to homogeneous media.

However, the solution for the particular case where **Injection** = **Conduction** = **Production** is not useful for unsteady reservoir conditions.

As a very simple example, although water cannot be **conducted** through an empty porous medium, there is no special impediment to **inject** water in the same porous system. An empty rock allows water **injection**. Additionally, although water can be **injected**, water cannot be **produced** until it reaches the outlet end.

As a consequence, in unsteady state systems there is not a "well defined" **conduction** ability, but there always exists a "well defined" **injection** ability and also a "well defined" **production** ability.

In reservoir simulation through Darcy´s calculations only **conduction** ability is used. For this reason Darcy´s equation is unable to reproduce **Injection** or **Production** rates in unsteady systems. And all reservoirs are unsteady systems during production.

Real Scenarios

In Reservoir Engineering:

- Natural porous media are heterogeneous.
- Multiphase flow is the result of equilibrium between viscous, capillary and gravity forces. This equilibrium varies with time and with physical location.
- Reservoir calculations are based on average phase saturation. This is the situation for a single cell (reservoir simulation) or the whole reservoir (material balance).
- The properties of interest are Production and Injection abilities.

Global Solutions

Every scenario has its own solution. Typical situations are:

- Homogeneous systems with dominant viscous forces. Very few reservoirs fall in this category, but it is the common laboratory setting.
- Heterogeneous systems with dominant viscous forces like stratified reservoirs without connected layers.
- Heterogeneous systems with dominant viscous and capillary forces like stratified reservoirs with connected

layers. Cross-flow occurs as a consequence of imbibition phenomena.

- Heterogeneous systems with heavy oil where imbibition phenomena could be dominant.
- Gravity dominated systems. Mainly in high permeability rocks with remarkable thickness and density differences and low viscosities (expanding gas cap, basal aquifers, etc.).
- Reservoirs dominated by capillary forces. Mainly in "tight sands" reservoirs.

Conclusions

Laboratory measurements are adequate to describe the ability to conduct fluids in porous media with homogeneous fluid saturation.

In reservoir engineering, the ability to **inject** or to **produce** fluids into media with non-homogeneous fluid saturation is needed. In unsteady systems the ability to **conduct** fluids loses its meaning and becomes useless.

To fill the gap, experimental measurements must be made to honor real mechanisms occurring at reservoir scale.

During the scaling-up stage, the following points must be considered:

- The dominant displacement mechanism.
- Laboratory "end point" measurements.
- System geometry.
- Heterogeneity.
- The scaling-up process must be a multidisciplinary team effort.
- Relative permeability curves must be "constructed" for every case. This operation must be based on available data and accepted reservoir models.