Chapter V

"... Under no circumstances, however, must the common practice of "averaging" relative permeability functions be attempted. In this, families of normalized curves, possibly from similar rock types throughout the field, are plotted together ... ... An average set of functions is then sought for water and oil either by "eye-balling" or using more sophisticated computer averaging procedures. The resulting functions are then input to all similar rock types in simulation models. Unfortunately, there is no physical principle that suggests that such a practice has any validity whatsoever. If any form of averaging were considered necessary (which it is not) then it would seem more appropriate to perform it on the fractional flow relationships which have a greater degree of physical reality - but even this is hard to justify. ..."

Dake, L.: "The Practice of Reservoir Engineering", Ed. Elsevier- p. 383

HETEROGENEITIES AND AVERAGES

Oil and gas are retained in widely heterogeneous traps. This fact is the result of the size of natural hydrocarbon reservoirs, the diverse conditions prevailing when the sedimentary material originating the rock was deposited, and the alterations suffered by this material over its whole geological history.

Natural heterogeneity is of such magnitude that even those formations described as "homogeneous" show significant changes in properties from one point to another.

The most significant property to characterize heterogeneity is perhaps rock permeability. It is not only highly related to the type of grains, sedimentary environment and diagenetic alterations, but it is also a property directly used to quantify fluid movement.

However, the direct measurement of rock properties is only possible at the few points where rock is extracted from subsoil. Thus, in order to obtain representative relative permeability curves, different rocks types are identified within the sedimentary structure, and laboratory measurements are made on each of them. This process, repeated in the few wells in which cores are extracted, leads to a limited set of relative permeability curves that must be used to characterize the entire reservoir.

Later, the reservoir engineer divides the reservoir into blocks or cells to which he must assign average properties. In general these blocks are heterogeneous. In fact they usually include several types of rocks, and therefore, several representative sets of relative permeabilities. This unavoidable situation has led to the practice of averaging relative permeability curves.

In brief, this chapter focuses on the validity of these average curves.

Once again, L Dake's introductory note [1] lets us to introduce the topic and, based on such a qualified introduction, the theoretical and practical procedure to obtain the average of relative permeability curves is analyzed.

Such analysis is quite relevant, since, in spite of formerly discussed limitations regarding the use of relative permeability curves, some kind of averaging is a very common practice.

> ***Note**: The concepts and procedures analyzed in this chapter apply to specific productivity curves (SPC). In addition, two supplementary subjects will be analyzed at the same time: sample selection for lab measurements and the advantages and disadvantages of the most common experimental methodologies to obtain relative permeability curves (steady and unsteady-state method).*

FLUIDS FLOW IN HETEROGENEOUS SYSTEMS

According to the general analysis style in this work, the study will be conducted on a very simple system, constituted by two horizontal superimposed layers, of equal thickness. The diagram of the model analyzed in this chapter is presented in Fig. V-1.

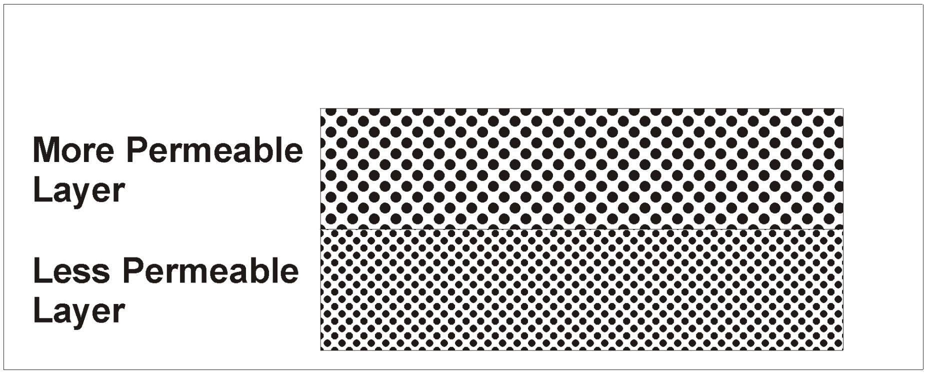

Fig. V-1: Cross-section of a two equal thickness parallel layer system. The analyzed block is horizontal.

To simplify the example, it is assumed that porosity is equal for both layers, while the thicker granulometry of the upper layer makes it 9 times more permeable than the lower layer.

It is also assumed that samples of both layers have been collected and after sending them to the lab, the experiments showed that both layers have exactly the same relative permeability curve (Fig. V-2).

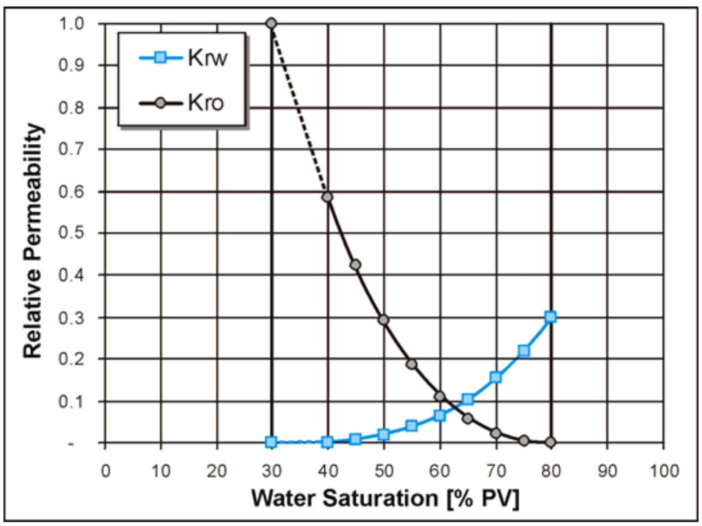

Fig. V-2: The same water-oil relative permeability curves for both layers in Fig. V-1.

The significant points of this curves, provided by lab measurements are:

- ✓ Swirr = 30 % PV.
- ✓ 100 – Sor = 80 % PV
- ✓ Kro[Swirr] = 1.00 (selected reference value)
- ✓ Krw[Sor] = 0.30.

***Note**: The same curves were chosen for both layers to simplify the subsequent calculations. In fact, there is no theoretical or empirical reason why two porous media -of different absolute permeabilities- would not have the same relative permeability curve. The shape of the relative permeability curves is more influenced by poral geometry (crosslinks, throat/poral diameter ratios, etc.) than by poral sizes. In other words, it can be said that by assigning the same relative permeability curve to both layers, we are assuming that they have similar poral geometry, but at different scale.*

The following question arises when simplifying the reservoir description, by using a homogeneous block of the same dimension as the one plotted in Fig. V-1:

What relative permeability curve should be assigned to a homogeneous block to reproduce the behavior of the heterogeneous block of Fig. V-1?

A simple analysis shows that optimal conditions are met to obtain the equivalent curve (or pseudo-curve) for the homogeneous block, since:

- ✓ The reservoir has only two layers.
- ✓ Each layer is fully homogeneous.
- ✓ Properties of each layer are fully known (geometry, thickness, porosity, permeability, relative permeability…).
- ✓ Relative permeability curves of both layers are identical.

In a first approach, if the arithmetical average of both curves were calculated, the conclusion would be that Fig. V-2 also describes the relative permeability curve of the full block in Fig. V-1.

In other words, since the block is constituted by two layers with identical relative permeability curves, any arithmetic or geometrical average of those curves produces a third identical curve.

Then, it seems pertinent to analyze if this is a satisfactory answer, or better still, **in which situations this answer becomes satisfactory**.

Case I. Unsteady-State Flow

Just to simplify calculations, it will be initially assumed that the layers are not communicated and mobility values for water and oil are equal (M = 1). Under these conditions, it is possible to make the calculations at a certain displacement point and verify whether the result is coincident with the forecasted result from curves in Fig. V-2.

Fig. V-3 shows the initial displacement, when the system is at Swirr conditions. Gray is used to represent oil as the only mobile fluid under initial conditions.

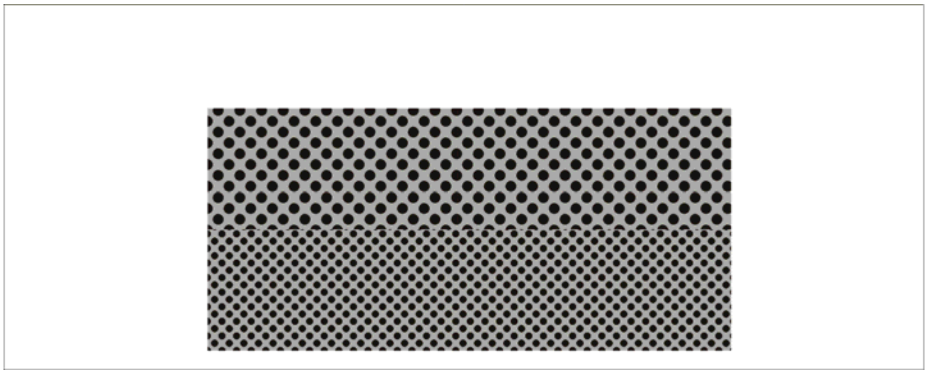

Fig. V-3: Initial status. Swirr in both layers. Only oil (in gray) is mobile.

Fig. V-4 shows an advanced stage of the water injection from the left end of the figure. Blue is used to indicate that oil has been displaced by water and only residual oil saturation (Sor) is left. Due to the assumed homogeneity for each layer and favorable fluids mobility, "perfect piston-like" displacement occurs, as shown in the scheme.

> ***Note**: It has already been mentioned in the preceding chapter, that trying to obtain "Perfect piston-like" displacement is an idealization, the only purpose of which is to simplify calculations. The conclusions to be drawn are qualitatively adequate for real displacements.*

There is no cross flow, since there is no communication between layers, therefore each layer is "swept" at its own rate.

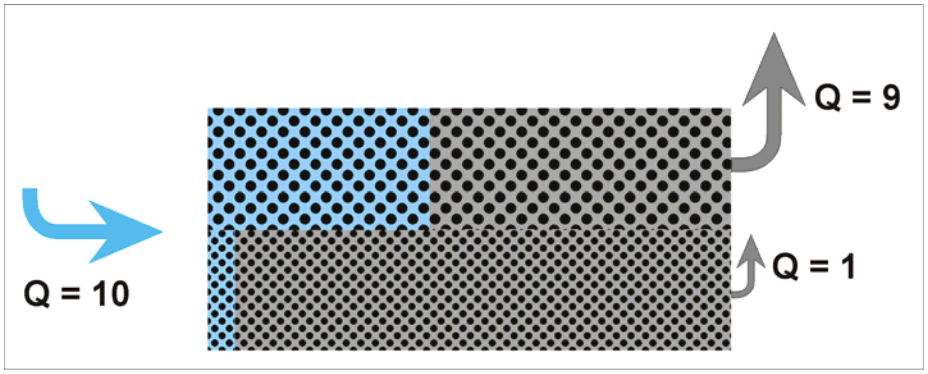

Fig. V-4 – Advanced stage of water injection from the left. Water (blue) displaces oil (gray) at different rates in each layer.

Injection rates are distributed proportionally to permeabilities at each level. The upper level receives, conducts and produces 90% of the total injected volume. At this stage, water is injected and only oil is produced, since “Breakthrough” has not yet been reached.

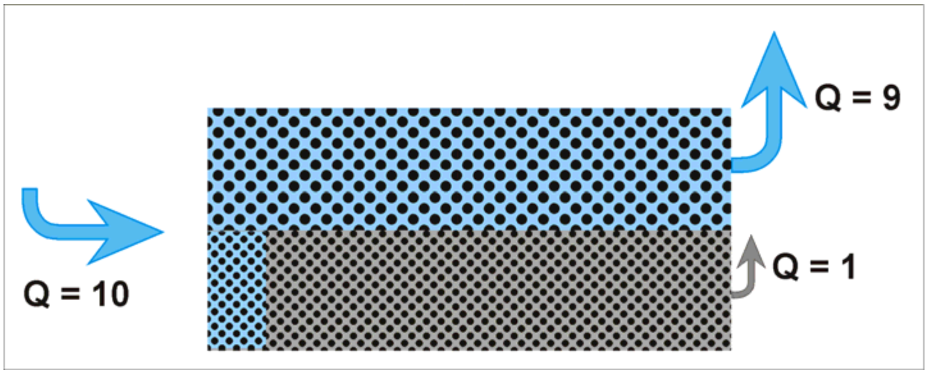

Fig. V-5 Water has reached the output face (Breakthrough). The less permeable layer still produces oil.

Fig. V-5 shows water reaching the output face in the most permeable layer. Simultaneous production of water (upper layer) and oil (lower layer) starts. This point is selected to calculate relative permeability values for water and oil.

Equal mobility between both phases guarantees that rate and differential pressure remain constant during the experiment, since there is no difference between the resistance offered by one phase or the other during displacement in the porous media. Thus, it is possible to make calculations of relative rates and phases saturation at the instant recorded in Fig. V-5.

Water Saturation (Sw)

In the output face, Sw gets the value corresponding to the average value between Swirr and (100-Sor). This is a direct calculation since both layers have the same porosity and thickness, one layer being at Swirr conditions while the other one at Sor conditions.

- ✓ Sw_{BT} = (30% PV+ 80% PV) / 2 = 55% PV.

In addition, average Sw for the entire system is not very different from this value because only the ninth part of the lower layer’s length has been flooded.

Relative Permeabilities

Since 9/10th of the flow rate takes place through the upper layer, at this point water production rate is equivalent to 9/10th of the rate to be reached when oil displacement is completed.

Thus, all other factors been equal, permeability becomes proportional to flow:

- ✓ Krw_{BT} = 0.30 x 9 / 10 = 0.27

A similar analysis shows that oil production at this point is only 1/10th of the initial oil production (at Swirr conditions).

- ✓ Kro_{BT} = 1.00 x 1 / 10 = 0.10

Summarizing the calculated values, the following points within the plot of relative permeability curves are obtained during an unsteady-state displacement dominated by viscous forces:

- ✓ **Sw** = (30% + 80%) / 2 = **55%**
- ✓ **Kro** = 1 * 1/10 = **0.10**
- ✓ **Krw** = 0.3 * 9/10 = **0.27**

When plotted, these points appear as shown in Fig. V-6.

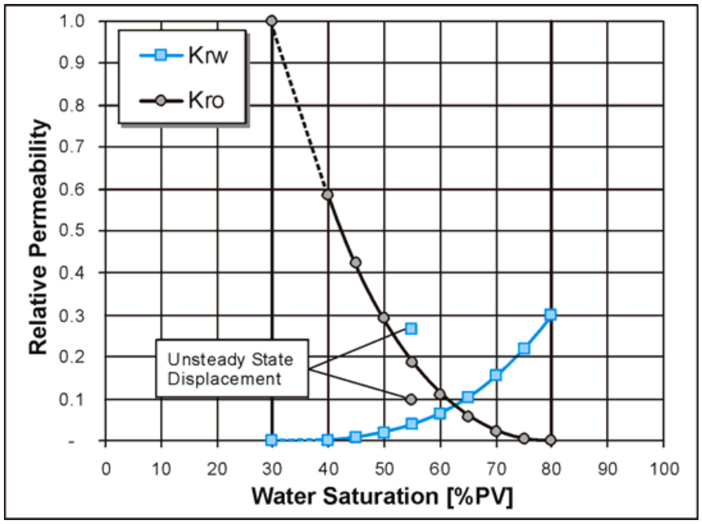

Fig. V-6: Relative permeability values calculated for unsteady-state displacement. Calculated points fall far apart from original relative permeability curves.

Discrepancy between these points and the curves obtained by means of the usual averaging procedures is quite noticeable. Water relative permeability increases from a value near 0.05 to 0.27, while oil relative permeability drops from 0.18 to 0.10.

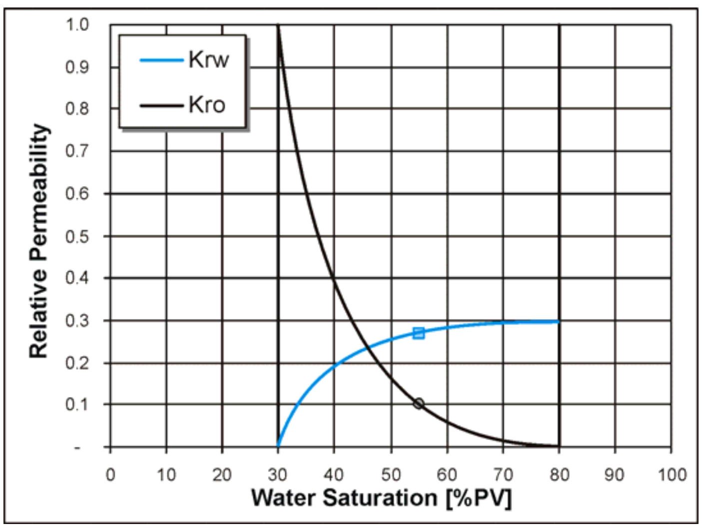

Fig. V-7: "Relative permeability curves" in unsteady-state displacement for the entire block in Fig. V-1.

Obtaining the full graph of the relative permeability curves for this block is not our present aim; however, Fig. V-7 shows the generic shape of relative permeability curves for markedly heterogeneous systems during unsteady-state displacement. As can be observed, this generic shape is compatible with calculated values.

> ***Note**: The curves plotted in Fig. V-7 are only schematic, they honor end points and values at water saturation (Sw=55%) where previous calculations had been carried out. Additionally, this type of curves was obtained during the analysis of the conceptual model developed in chapter I (Fig. I-10).*

Conceptually, the shape taken by relative permeability curves in heterogeneous systems, during an unsteady-state displacement originates in an easy-to- describe situation:

- ✓ A fraction of the sample is always swept more quickly than the remainder of the system in heterogeneous porous media.
- ✓ Once the most permeable zone is swept, Sw grows only in proportion to this zone's poral volume, but relative permeability to water increases noticeably, precisely because this is the most permeable zone. It can be said that, for heterogeneous systems with parallel layers, relatively small increments in Sw cause a strong increase in water relative permeability.

Summary

The previous development may be summarized as follows:

- ✓ A very simple model with two different permeability layers was selected.
- ✓ Equal values were given to the remaining petrophysical properties of both layers
- ✓ Relative permeability values of this model were calculated –in a simplified way- at a given water saturation point, during an unsteady- state displacement.
- ✓ The resulting relative permeability values are very different from the values taken by the relative permeability curves previously obtained for each layer, at the same Sw. These values would not be obtainable through any simple averaging calculation technique.
- ✓ The end points of the system definitely can be obtained by a simple average technique using the values for each layer.
- ✓ Capillary or gravity force effects during the process were not taken into account.
- ✓ Dake's expression, quoted at the beginning of this chapter, appears to be fully applicable to these displacements.

Remarks

Once again, it is important to remember that this development only takes into account the action of viscous forces. This situation is consistent with laboratory routine measurement of relative permeabilities curves.

It is not necessary to assume no communication between layers when working with identical mobility for both phases (M = 1). In this case (predominant viscous forces), since gradients remain linear and identical for both levels, cross flow does not occur even in the case of communicated layers.

Case II. Steady-State Flow

In steady-state displacement, both mobile phases are injected at a fixed rate until production becomes identical to injection. The steady-state situation is reached after overcoming a transient period (unsteady-state situation). In other words, the measurement and calculation sequence is as follows:

1. A flow ratio for both phases is arbitrarily chosen, for instance: Qw/Qo = 0.20.
2. Injection is kept at this fixed rate until the production rates match exactly the injected ones. Steady state is reached at this point because the system does not experience any additional changes (system input is identical to system output).
3. Once the system is in steady state, Darcy equation is applied to obtain the effective permeabilities to each phase. Phase saturation, which is automatically established as a result of the selected flow ratio, is measured separately.
4. The set of Sw, Kro and Krw values determines a point in the relative permeability curves of the system under study.
5. The fractional flow of water is increased and the whole sequence is repeated from point "1" on until the complete curve is built.

The study can be made at any point in the curve. Therefore, the following fluids and injection rates are arbitrarily chosen:

- ✓ μ_w = 1 cp
- ✓ μ_o = 1 cp
- ✓ Qw = 0.2 cm^3/min
- ✓ Qo = 1.0 cm^3/min
- ✓ Qw/Qo= 0.2

As viscosities are identical, the flow ratio becomes proportional to the relative permeability ratio. Fig. V-8 shows that the selected relative permeability ratio (0.2) is established when Sw=55 %.

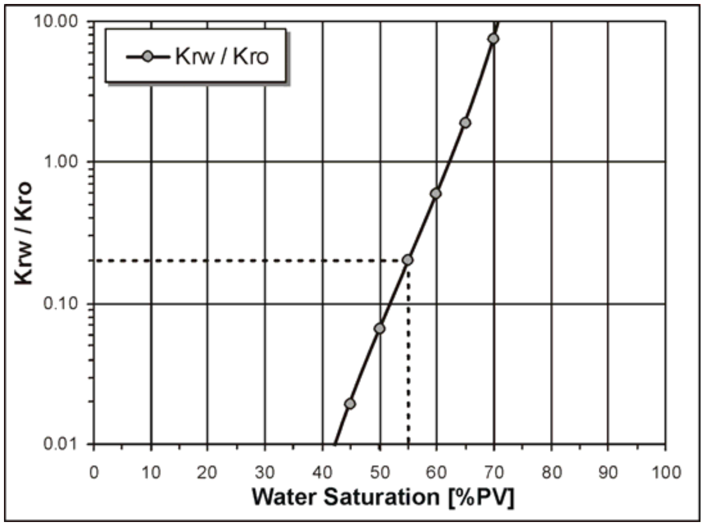

Fig. V-8: Relative permeabilities ratio corresponding to "experimental" curves in Fig V-2. Krw / Kro = 0.2 ratio is obtained for Sw = 55% PV

Thus, steady-state condition (identical Sw in the entire porous medium) will be reached in any of the layers where such fluid proportion is injected if Sw value becomes 55 % PV. At such saturation point, the flow ratio circulating in the porous media (Qw / Qo = 0.2) is determined by the same value in the relative permeability ratio.

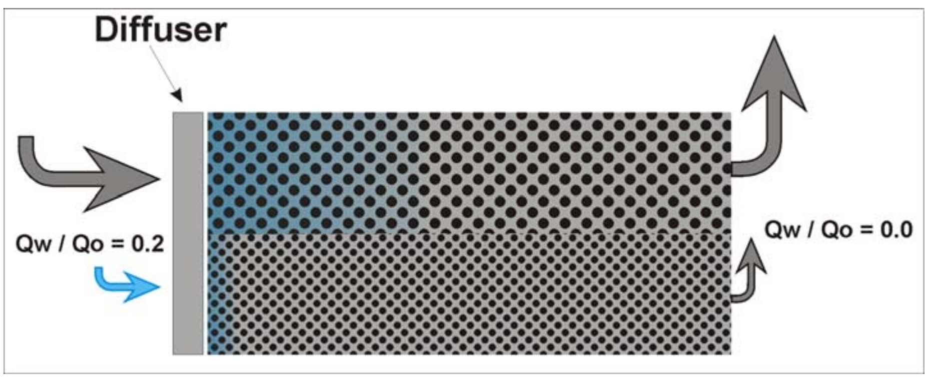

Fig. V-9: Both layers receive water and oil but at a rate proportionally to its own permeability.

In other words, when steady-state condition is reached, the water saturation at any point and the average water saturation in each layer become equal, and the value for both is 55%.

The above analysis is plotted in the following figures.

Fig.V-9 shows the saturation and production situation shortly after the simultaneous injection of water (blue) and oil (gray) has begun using the previously mentioned rate proportion.

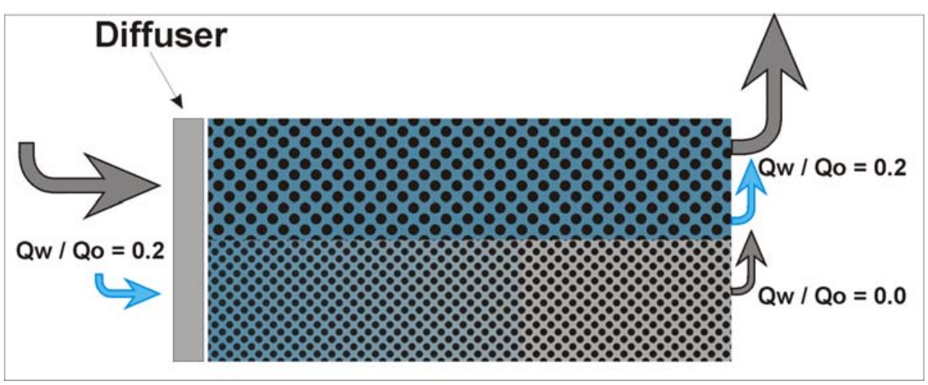

Fig V-10: The most permeable layer reaches steady state while the less permeable layer is still in transient state.

A diffuser (homogeneous porous medium) is added to the diagram, the function of which is to distribute both phases homogeneously all over the fluids input face.

Fig.V-10 shows water injection at a more advanced state. The more permeable layer has already reached steady state (production ratio identical to injection ratio). The less permeable layer is still in "transient" state so the whole system has not reached steady state yet.

Fig.V-11 shows the system once steady state is reached. On this case, production ratio in both layers (Qw / Qo = 0.20) is identical to injection ratio. Each layer contributes to total production proportionally to its permeability.

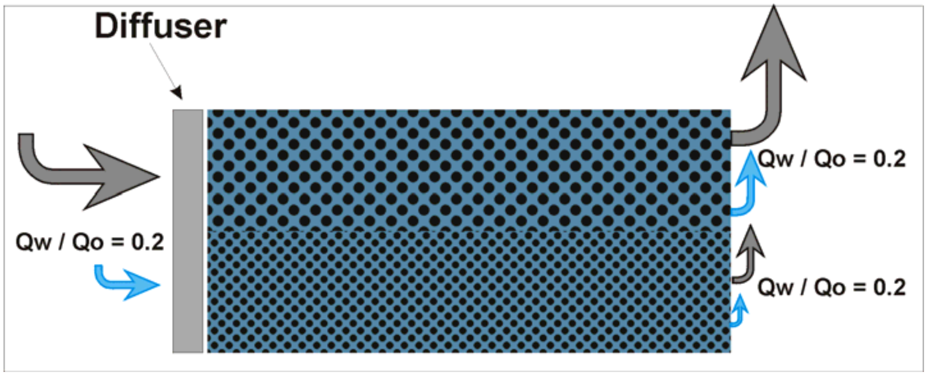

Fig V-11: Both layers at steady state condition and producing proportionally to their permeabilities ratio.

Making the calculations for the steady-state condition and taking into account that 9/10th of total rate circulate in the upper layer, the result indicated in Table V-1 is obtained:

Table V – 1
Steady State Relative Permeabilities

	Sw [%PV]	**Qo [cm^3/min]**	**Qw [cm^3/min]**	**Qw/Qo**
Upper Layer	55	1.0 x 0.9 = 0.90	0.2 x 0.9 = 0.18	0.20
Lower Layer	55	1.0 x 0.1 = 0.10	0.2 x 0.1 = 0.02	0.20
Fig. V-1 Entire System	55	0.90 + 0.10 = **1.0**	0.18 + 0.02 = **0.20**	0.20

As can be easily noticed, Relative permeability values of the entire system are the same as those of layers individual layers.

> ***Note 1**: In this example, the development was made for a given point only; it was demonstrated that flow ratios and fluid saturation in the entire system are identical to those in each layer. By repeating the analysis for other injection ratios, it can be demonstrated that the curve in Fig. V-2 also describes the behavior of the heterogeneous system showed in Fig. V-1 in the whole range of mobile saturations*

> ***Note 2**: With two layers having different relative permeability curves, the same procedure may be applied to demonstrate that the curve for the heterogeneous block results to be the arithmetical average of both curves, with weighting factors proportional to the effective PV's of each layer.*

Summary

The above development may be summarized as follows:

- ✓ A very simple model with two different permeability layers was selected.
- ✓ Equal values were given to the remaining petrophysical properties of both layers
- ✓ Relative permeability values of this heterogeneous model were calculated –in a simplified way- at a given water saturation point, during a steady-state displacement.
- ✓ The relative permeability values thus obtained are equal to the values for each individual layer.
- ✓ The end points of the system can also be obtained by a simple average technique using the values for each layer.
- ✓ Capillary or gravity force effects during the process were not taken into account.
- ✓ Dake's[1]' expression, quoted at the beginning of this chapter, does not seem applicable to steady-state displacements.

Comments

1. It is important to remember that only viscous forces were taken into account in this development. This situation is consistent with laboratory routine relative permeability tests.
2. Once again, it is not necessary to initially assume no communication between layers. Under steady state viscous flow conditions, cross flow does not occur between communicated layers.

ANALYSIS OF RESULTS

In the previous developments, two different methodologies were analyzed to obtain the average of the relative permeability curves in a heterogeneous system. As it could be appreciated, the results of steady-state and unsteady-state averages are quite different. For such reason, their validity and applicability is discussed below.

Mathematical Average

The main results of the two developments can be summarized as follows:

1. Relative permeability curves as calculated for unsteady-state flow are very different from those curves corresponding to each layer. No simple averaging technique may be applied.
2. Relative permeability curves as calculated for steady-state flow are equal to the arithmetical average of the curves corresponding to each layer.
3. In both cases, the end points of the system respond to an arithmetical average. This item may be derived from item "2", since steady-state flow conditions (single phase injection and production), are met at end point saturations.

Taking into account that porous medium heterogeneity results from different poral structures, and assuming that each of these structures has its own relative permeability curve, we may further summarize the overall conclusions:

Steady-state method is the only one that automatically generates a direct average of relative permeability curves corresponding to each structure in a heterogeneous porous medium.

In fact, the above expression is the main argument used to prefer lab measurements using the steady-state method.

Physical Average

The previous argument shows the adequate solution from the viewpoint of mathematical treatment of results. But…, when the behavior of real reservoirs is analyzed, conclusions may be quite different.

Valid conclusions for real reservoir characterization are as follows:

1. Viscous displacements (the only ones here analyzed) are unsteady-state at reservoir scale. In general, while one phase is injected (frequently water or gas) a differente one is produced (oil). In fact, in obvious contrast to steady-state measurement, attempts are made to maximize the difference between what is injected and what is produced.
2. If lab samples are heterogeneous, the reservoir is certainly heterogeneous as well.

Since the end points obtained through unsteady-state flow are identical to those measured in steady-state flow conditions, the following question arises: *What is the possible application of the curves obtained by the steady-state method, at reservoir scale?*

Or, in a more generic way:

What are the possible applications, at reservoir level, of curves obtained by arithmetic average of those curves corresponding to the structures responsible of system's heterogeneity?

And the answer to this question is conclusive. All previous analysis lead to full coincidence with L. Dake's[1] phrase, quoted at the beginning of this chapter, particularly with the expression:

> *"... Under no circumstances, however, must the common practice of 'averaging' relative permeability functions be attempted ..."*

However, this expression has to be taken in a wider sense, making also reference to experimental methodologies used to obtain relative permeability curves. In fact, reasons justifying steady state lab measurement are really hard to find.

Results of steady-state measurements of relative permeability curves are not applicable at reservoir scale. Only end points are of interest, but they are coincident with those obtained through unsteady-state techniques.

A Conceptual Answer

Based on the developments in this chapter, the generic answer to the question *How should Relative Permeability Curves be averaged?* is given below.

- ✓ The mathematical averaging of Relative Permeability curves means absolutely nothing from the physical point of view since, in heterogeneous systems, the property of interest is the production capacity of each phase as a function of the average saturation of the system. And this curve, besides being conditioned by the individual relative permeability curves (in fact, by the specific productivity curves) receives the dynamic influence of many additional factors such as:
  - Effective permeabilities. They determine the flooding sequence of the structures that define the system's heterogeneity.
  - Heterogeneity distribution. When predominant forces are other than viscous forces, displacement is affected by the relative location of the permeable levels.
  - Viscosity ratio. This ratio influence viscous cross flow.
  - Equilibrium of forces. Which is directly affected by exploitation conditions (mainly injection / production rate).

Then, the solution to be used depends on each reservoir and its exploitation conditions. The conceptual analysis of different scenarios is deferred to Chapter VII.

> ***Note***: *In this chapter, only a partial analysis was made. As discussed in the Appendixes, at the end of this book, if flow is not parallel to the layers, or if displacement occurs under capillary and/or gravity forces, end points saturation are usually different[2] from those obtained through viscous displacements parallel to stratification planes.*

FAQs AND COMMENTS

As in other chapters, it is advisable to include some typical questions and comments, generated during the explanation of the topics here developed.

Question: If it has no application, why is averaging of relative permeability curves such a common practice?

Answer: It is very difficult to answer this question. In the author's opinion, this practice, as many practices related to the use of relative permeabilities, was developed to simplify physical reality having in mind the limitations of available models. Once on this path, the physical objective is frequently lost, giving priority to numerical or mathematical objectives.

Question: What other practices show similar trends?

Answer: The author has observed a strongly tendency to attempt to **always** find a unique set of relative permeability curves. This applies to lab measurement (where all the variables leading to different answers are eliminated) and to reservoir applications, where attempts are made to find "**the curve**" describing any system, however complex it is. This tendency does not take into account the physical reality which usually shows production capacity noticeably dependent on displacement conditions.

Question: Then, isn't it possible to simplify real models to make them more "friendly"?

Answer: Just the opposite. They can be noticeably simplified. In fact, several chapters of this book draw attention to the fact that widely used methodologies and concepts trying to get this simplification are not adequate for such objective.

Question: What methodologies and concepts are you referring to?

Answer: Among others, we can highlight two of them.

- ✓ The (implicit or explicit) belief in the uniqueness of relative permeability curves. This belief is related to the "veneration" of these curves, as quoted from L. Dake already al the beginning of this book.
- ✓ The belief that relative permeability curves (developed to describe fluid conduction) are adequate to describe production phenomena. This belief cannot be supported on any real case.

Question: Conclusions presented so far suggest that relative permeability measurements through steady-state method are unnecessary. Is it true?

Answer: They are not only unnecessary. Their results are not applicable to describe real reservoirs behavior.

Question: However, as mentioned in this book, the steady-state test was developed before the unsteady-state methodology. And many specialists consider it the "standard" method to obtain relative permeabilities.

Answer: Right. However, it should be mentioned that the steady-state method was essentially developed to determine whether Darcy's equation was valid in multiphase flows. In other words, it was born as a conceptual or qualitative method. Darcy's conditions can only be recreated in steady state conditions, since the equation parameters (geometry, viscosity, rate, pressure difference) are perfectly defined for each phase, for a given saturation. Its later use to solve real (dynamic and unsteady state) cases is a conceptual error extended over time.

Question: From the above, may it be deduced that only saturation and permeability end points are applicable in real cases?

Answer: Yes, but with some reservations. In the Appendixes to this book we analyze end points dependence on displacement conditions.

Question: Since heterogeneity affects dynamic measurement results, how should lab samples be chosen?

Answer: On this subject, statistical and geological representative criterion has to be met. In most cases it is impossible to obtain homogeneous samples from each significant layer. Usually heterogeneities of each individual layer are evident even at lab sample scale. In this case, it is advisable to remember that the layer cannot be more homogeneous than the samples obtained from it

SUMMARY AND CONCLUSIONS

Since this chapter has simply validated L. Dake's phrase quoted at the beginning, it is advisable to re-read such phrase as summary and conclusion.

REFERENCES

1. Dake, L.: "*The Practice of Reservoir Engineering*", Ed. Elsevier - Pág. 383
2. Crotti, M. A., Cobeñas, R. H., "*Puntos Extremos de Saturación. Medición en Laboratorio y Traslado de la Información al Reservorio*", Presented at IAPG 2000 Production Congress. Cataratas del Iguazú, May 8-12, 2000.